

Benha University Faculty of Engineering – Shoubra Department of Industrial Engineering Course: Mathematics 2 Code: EMP 102		Final Exam Date : January 21, 2017 Answer All questions Duration: 2 hours
• The exam consists of one page	• No. of questions: 4	Total Mark: 40
<u>Question 1</u>		
Find y' from the following:		
(a) $y = 4^x + 3 \sinh x$	(b) $y = (x^3 + \cosh x)^5$	(c) $y = \sin^{-1} x + \sinh^{-1} x$
(d) $y = \ln x + \log x$	(e) $y = 8 + x \cdot \tanh x$	(f) $y = e^y + x \cdot \cosh x$
<u>Question 2</u>		
Find the integrals:		
(a) $\int (x^3 - 4^x + 4) dx$	(b) $\int (\sinh 2x - \cosh x) dx$	(c) $\int \frac{x+1}{x^2-6x+8} dx$
(d) $\int \ln x dx$		
<u>Question 3</u>		
(a) Find the area of the region between the curve $y = x^2 - 1$, x-axis, x in $[0, 2]$.		
(b) If the region between the curve $y = x^2 + 2$, x-axis, x in $[1, 2]$ is rotated about		
(i) x-axis (ii) y-axis. Find the volume of the generated solids V_x , V_y .		
(c) Separate the lines and find the angle between them : $x^2 - 6xy + 8y^2 = 0$.		
<u>Question 4</u>		
(a) State the definition of the circle.		
(b) State the definition of the ellipse.		
(c) Write the equation of circle where the points $(1, 2)$, $(3, -1)$ are ends of diameter.		
(d) Find the vertex, focus and sketch the parabola : $y^2 + 8x - 4y - 20 = 0$.		
(e) Find center, vertices and sketch the hyperbola : $4x^2 - 9y^2 - 8x + 36y + 4 = 0$.		

Good Luck

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Model Answer

Answer of Question 1

$$(a) y' = 4^x \cdot \ln 4 + 3 \cosh x$$

$$(b) y' = 5(x^3 + \cosh x)^4(3x^2 + \sinh x)$$

$$(c) y' = \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1+x^2}}$$

$$(d) y' = \frac{1}{x} + \frac{1}{\ln 10} \cdot \frac{1}{x}$$

$$(e) y' = 0 + 1 \cdot \tanh x + x \operatorname{sech}^2 x$$

$$(f) y' = e^y \cdot y' + 1 \cdot \cosh x + x \sinh x$$

-----6- Marks

Answer of Question 2

$$(a) \int (x^3 - 4^x + 4) dx = \frac{x^4}{4} - \frac{4^x}{\ln 4} + 4x + c$$

$$(b) \int (\sinh 2x - \cosh x) dx = \frac{1}{2} \cosh 2x - \sinh x + c$$

$$(c) \int \frac{x+1}{x^2-6x+8} dx = \int \left(\frac{5/2}{x-4} - \frac{3/2}{x-2} \right) dx = \frac{5}{2} \ln(x-4) - \frac{3}{2} \ln(x-2) + c$$

$$(d) \int \ln x dx = x \ln x - x + c$$

-----12- Marks

Answer of Question 3

(a) From $x^2 - 1 = 0$, we get $x = 1$ inside the interval and $x = -1$ outside the interval

$$\text{Then } A = \int_0^1 (x^2 - 1) dx + \int_1^2 (x^2 - 1) dx = \left| -\frac{2}{3} \right| + \frac{4}{3} = 2$$

-----3- Marks

$$(b)(i) V_x = \pi \int_1^2 (x^2 + 2)^2 dx = 19.53\pi$$

$$(ii) V_y = 2\pi \int_1^2 x(x^2 + 2) dx = 13.5\pi$$

-----6- Marks

$$(c) x^2 - 6xy + 8y^2 = (x - 2y)(x - 4y) = 0$$

$$\text{The two lines are : } x - 2y = 0, \quad x - 4y = 0 \quad \text{and} \quad \tan \theta = \frac{\frac{1}{2} - \frac{1}{4}}{1 + \frac{1}{8}} = -\frac{2}{9}$$

-----2- Marks

Answer of Question 4

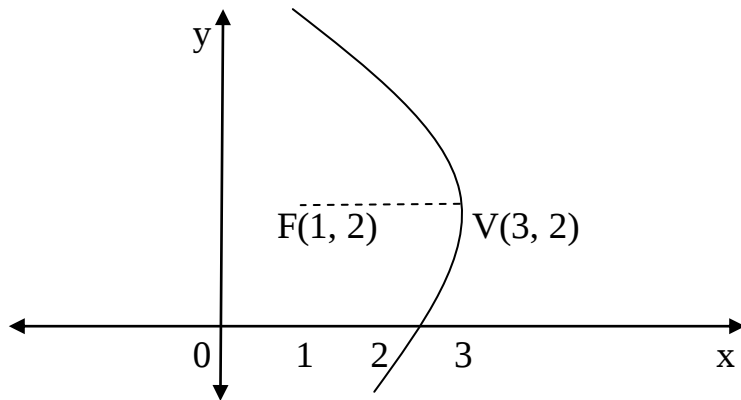
(a) Circle

(b) Ellipse

-----2- Marks
(c) The circle is : $(x - 1)(x - 3) + (y - 2)(y + 1) = 0$ Or $x^2 + y^2 - 4x - y + 1 = 0$

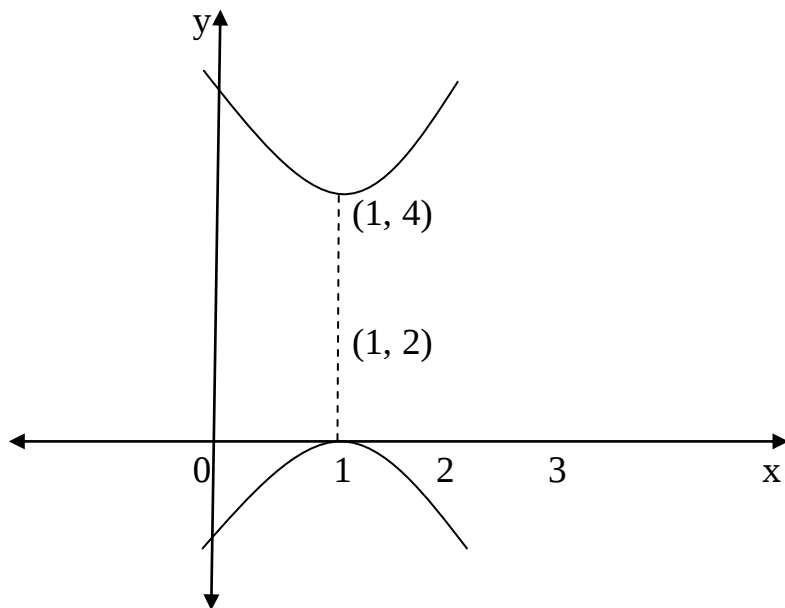
-----3- Marks
(d) From : $y^2 + 8x - 4y - 20 = 0$, we get $(y - 2)^2 = -8(x - 3)$

The vertex is $(3, 2)$, $a = -2$.



-----3- Marks
(e) From : $4x^2 - 9y^2 - 8x + 36y + 4 = 0$, we get $\frac{(y-2)^2}{4} - \frac{(x-1)^2}{9} = 1$

The center is $(1, 2)$, $b = 2$.



-----3- Marks

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